Mark Scheme (Results)

Summer 2024

Pearson Edexcel GCE
In A Level Further Mathematics (9FM0)
Paper 4C Further Mechanics 2

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $*$ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
   If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.

- Omission or extra g in a resolution is an accuracy error not method error.

- Omission of mass from a resolution is a method error.

- Omission of a length from a moments equation is a method error.

- Omission of units or incorrect units is not (usually) counted as an accuracy error.

- dM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.

- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.

- Use of g = 9.81 should be penalised once per (complete) question.
  N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

- Marks must be entered in the same order as they appear on the mark scheme.

- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.

- Accept column vectors in all cases.

- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

- Mechanics Abbreviations
  M(A) Taking moments about A
  N2L Newton's Second Law (Equation of Motion)
  NEL Newton's Experimental Law (Newton's Law of Impact)
  HL Hooke's Law
  SHM Simple harmonic motion
  PCLM Principle of conservation of linear momentum
  RHS, LHS Right hand side, left hand side

Question	Scheme	Marks	AOs
1a	$\int \frac{96}{(3t+5)^3}\,\mathrm{d}t = \int 1\,\mathrm{d}v \quad \Rightarrow v = ...$	M1	2.1
	$\Rightarrow -\frac{96}{2\times 3\times (3t+5)^2}(+C) = v$	A1	1.1b
	Use limits $v = 0, t = 0$	M1	1.1b
	$\Rightarrow v = \frac{96}{6\times (5)^2} - \frac{96}{6\times (3t+5)^2} = \frac{16}{25} - \frac{16}{(3t+5)^2}$ *	A1*	2.2a
		(4)	
1b	$t \to \infty \Rightarrow v \to \frac{16}{25}(= 0.64)$	B1ft	2.2a
		(1)	
1c	$\int 1\,\mathrm{d}x = \int \frac{16}{25} - \frac{16}{(3t+5)^2}\,\mathrm{d}t \Rightarrow x = rt + s\frac{1}{3t+5}$	M1	2.1
	$x = \frac{16}{25}t + \frac{16}{3(3t+5)}(+D)$	A1ft	1.1b
	$x = \left[\frac{16}{25}t + \frac{16}{3(3t+5)}\right]_0^2$	M1	1.1b
	$x = \left(\frac{32}{25} + \frac{16}{3(11)}\right) - \left(\frac{16}{3(5)}\right) = \left(\frac{192}{275}\right) = 0.70$ or better	A1	2.2a
		(4)	
			Total 9 marks

Notes:

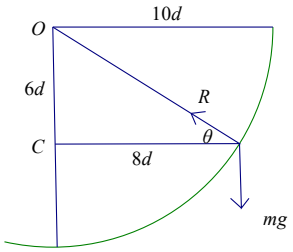
1a	M1	Form a differential equation in v and t and integrate. Must attempt integration of $\frac{k}{(3t+5)^3}$. RHS can be implied.
	A1	Correct integration. Ignore any limits. Accept without constant of integration.
	M1	Use $v = 0, t = 0$ as limits in a definite integral or to find the constant of integration
	A1*	Obtain given answer in the form $v = p - \frac{q}{(3t+5)^2}$ from correct working. Accept if correct form given and values of p and q stated separately. Must have "$v =$".
1b	B1ft	Follow through their p

1c	M1	Form a differential equation in x and t and integrate to obtain $rt + s\dfrac{1}{3t+5}$ where r and s are rational
	A1ft	Correct integration. Ignore limits and condone no constant of integration. Follow through their p and their $-\dfrac{q}{3}$
	M1	Use $x = 0, t = 0$ as limits in a definite integral or substituted to find the constant of integration **and** find x when $t = 2$
	A1	0.70 or better. (0.698181…)

Question	Scheme	Marks	AOs
2(a)	Moments about ED	M1	2.1
	e.g. $4\times 4a\times 2a\cos 30° + 2\times 4a\times 4a\cos 30° = 7\times 4a\times d$	A1 A1	1.1b 1.1b
	$32\sqrt{3}a^2 = 28ad \Rightarrow d = \frac{8\sqrt{3}}{7}a$ *	A1*	1.1b
		(4)	
2(b)	Moments about C	M1	3.1a
	$8a\times F = \left(4a\cos 30° - \frac{8\sqrt{3}}{7}a\right)\times W$	A1	1.1b
	$F = \frac{3\sqrt{3}}{28}W$	A1	1.1b
		(3)	
			Total 7 marks

Notes:

2a	M1	Dimensionally correct equation with required terms. Accept use of a parallel axis. Accept equivalent mass ratio e.g. $4a$ replaced by 1
	A1 A1	Unsimplified equation with at most one error. Allow distances in terms of $\sin 60°$ or $\cos 30°$ or equivalent. N.B. Repeated use of an incorrect distance is only one error. Correct unsimplified equation. Allow distances in terms of $\sin 60°$ or $\cos 30°$ or equivalent
	A1*	Obtain given answer from correct working including reference to d.
2b	M1	Dimensionally correct equation with required terms and no extras Equation should be of the form $\lambda F = (\mu - d) W$
	A1	Correct unsimplified equation
	A1	$0.19W$ or better (0.185576….W)

Question	Scheme	Marks	AOs
3a			
	Resolve vertically	M1	3.3
	$R\sin\theta = mg$	A1	1.1b
	Horizontal equation of motion	M1	3.3
	$R\cos\theta = ma\left(=mr\omega^2\right)$	A1	1.1b
	Solve for a	DM1	2.1
	$\frac{g}{a} = \tan\theta \Rightarrow a = \frac{4}{3}g$	A1	1.1b
		(6)	
3b	$8d\omega^2 = \frac{4}{3}g \Rightarrow \omega = \sqrt{\frac{g}{6d}}$	M1	3.4
	Use of $T = \frac{2\pi}{\omega}$	M1	1.1b
	$T = 2\pi\sqrt{\frac{6d}{g}}$ oe OR $15\sqrt{\frac{d}{g}}$ or better	A1	1.1b
		(3)	
			Total 9 marks

Notes:

3a	M1	Dimensionally correct equation. Condone sine / cosine confusion for their angle
	A1	Correct unsimplified equation
	M1	Dimensionally correct equation. Condone sine / cosine confusion for their angle. Accept any correct form for the acceleration.
	A1	Correct unsimplified equation. Accept any correct form for the acceleration.
	DM1	Eliminate R and θ to solve for a. Dependent on both previous M marks.
	A1	Correct only (or exact equivalent)
3b	M1	Use $a = r\omega^2$ or $a = \frac{v^2}{r}$ to obtain ω or v $\left(v = \sqrt{\frac{32dg}{3}}\right)$

	M1	Complete method to find T
	A1	$15\sqrt{\frac{d}{g}}$ or better - must be in terms of d and g. Accept $T = \frac{2\pi}{\sqrt{\frac{g}{6d}}}$

Question	Scheme	Marks	AOs
4a	Moments about the y-axis: $\int(\rho)xy\,\mathrm{d}x$	M1	3.1a
	$=(\rho)\int x\sqrt{36-\frac{9x^2}{4}}\,\mathrm{d}x=k\left(36-\frac{9x^2}{4}\right)^{\frac{3}{2}}$	M1	2.1
	$=\left[-\frac{4}{27}\left(36-\frac{9x^2}{4}\right)^{\frac{3}{2}}(\rho)\right]_0^4\left(=32(\rho)\right)$	A1	1.1b
	$\bar{x}=\frac{\int xy\mathrm{d}x}{6\pi}$	DM1	3.1a
	$\bar{x}=\frac{32}{6\pi}=\frac{16}{3\pi}$ *	A1*	2.2a
		(5)	
4b	Moments about the x-axis: $\int\frac{1}{2}y^2(\rho)\mathrm{d}x\left(=\frac{1}{2}(\rho)\int 36-\frac{9x^2}{4}\mathrm{d}x\right)$	M1	3.1a
	$=\frac{1}{2}(\rho)\left[36x-\frac{3}{4}x^3\right]_0^4\left(=\frac{1}{2}(\rho)(144-48)=48(\rho)\right)$	A1	1.1b
	$\bar{y}=\frac{\int\frac{1}{2}y^2\mathrm{d}x}{6\pi}$	DM1	2.1
	$=\frac{48}{6\pi}\quad\left(=\frac{8}{\pi}\right)$	A1	2.2a
		(4)	
4c	Correct use of trigonometry	M1	3.1a
	$\tan\theta°=\frac{\textit{their}\ \bar{y}}{4-\frac{16}{3\pi}}\left(=\frac{6}{3\pi-4}\right)$	A1ft	1.1b
	$\theta=47.9\quad(48\text{ or better})$	A1	1.1b
		(3)	
			Total 12 marks

Notes:		
4a	M1	Correct method for moments about the x-axis: $\int(\rho)xy\,\mathrm{d}x$ or $\int(\rho)\frac{1}{2}x^2\,\mathrm{d}y$ Integrand should be in one variable only.
	M1	Integrate to obtain $k\left(A-Bx^2\right)^{\frac{3}{2}}$ Ignore limits and / or constant of integration

	A1	Correct integration with correct limits seen or implied.
	DM1	Complete method to obtain $\bar{x}$. Dependent on **first** M1.
	A1*	Obtain given answer from correct working
4b	M1	Correct method for moments about the y-axis: $\int(\rho)xy\,\mathrm{d}y$ or $\int(\rho)\frac{1}{2}y^2\,\mathrm{d}x$
	A1	Correct integration with correct limits seen or implied.
	DM1	Complete method to obtain $\bar{y}$. Dependent on previous M1.
	A1	Correct exact equivalent
4c	M1	Correct use of trigonometry to find a relevant angle
	A1ft	Correct unsimplified expression for tan θ or its reciprocal. Follow their $\bar{y}$
	A1	48 or better $(47.8823...)$ 0.836 radians is A0

Question	Scheme	Marks	AOs
5a	$a\omega^2 = 18$	B1	3.4
	Use $v^2 = \omega^2\left(a^2 - x^2\right)$	M1	3.4
	$2.4^2 = \omega^2\left(a^2 - 0.3^2\right)$	A1	1.1b
	Form equation in a only and solve for a e.g. $\frac{a}{a^2 - 0.09} = \frac{18 \times 25}{144}$ $\left(18a^2 - 5.76a - 1.62 = 0\right)$	M1	3.1a
	$\Rightarrow a = 0.5$ *	A1*	1.1b
		(5)	
5b	Solve for ω and use max speed $= a\omega$	M1	3.4
	Greatest speed $= 0.5 \times 6 = 3\left(\text{ms}^{-1}\right)$	A1	1.1b
		(2)	
5c	$\lvert v\rvert = a\omega\sin\omega t$ **OR** $v^2 = \omega^2\left(a^2 - x^2\right)$ and $x = a\sin\omega t$	B1ft	3.3
	$\Rightarrow 2 = 3\sin 6t$ $\left(t = 0.1216.....\right)$ **OR** $\frac{\sqrt{5}}{6} = 0.5\sin 6t$ $(t = 0.14017\ldots)$	M1	1.1b
	$S = \frac{2\pi}{6} - 4t$ **OR** $S = 4 \times 0.14017\ldots$	M1	3.1a
	$= 0.5607....$	A1	1.1b
		(4)	
		Total 11 marks	

Notes:

5a	B1	Use the model to state correct equation for greatest acceleration. Accept ±18.
	M1	Use the model to form a second equation in a and ω
	A1	Correct unsimplified equation with x and v substituted.
	M1	Solve the simultaneous equations to obtain a
	A1*	Correct only from **correct** working. Must have used positive ω^2.
5b	M1	Complete method using the model to obtain greatest speed E.g. $v = \sqrt{\omega^2\left(0.5^2 - 0^2\right)}$
	A1	Correct only
5c	B1ft	Set up a correct model to find time when speed is 2. ft their ω (If starting from O then $\lvert v\rvert = a\omega\cos\omega t$)

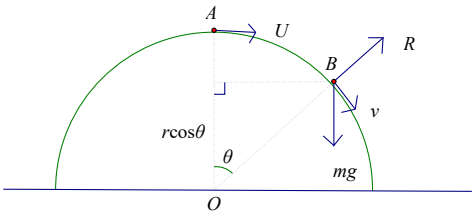
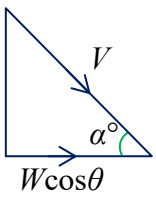
	M1	Solves their equation(s) to obtain a critical value for t $\left(2 = 3\cos 6t \Rightarrow t = 0.14017...\right)$ (condone degrees: 41.8°/48.2°)
	M1	Correct method to obtain S; must be in radians $\left(S = 4 \times 0.14017...\right)$
	A1	0.56 or better

Question	Scheme	Marks	AOs
6a	Mass $= \int_0^6 \pi 2^2 \lambda (x+2)\,\mathrm{d}x$	M1	3.4
	$= 4\pi\lambda\left[\frac{x^2}{2}+2x\right]_0^6$	A1	1.1b
	$= 4\pi\lambda\left(\frac{36}{2}+12\right) = 120\lambda\pi\,(\text{kg})$ *	A1*	2.2a
		(3)	
6b	Moment about y-axis $= \int_0^6 4\pi x\lambda(x+2)\,\mathrm{d}x$	M1	2.1
	$= 4\pi\lambda\left[\frac{x^3}{3}+x^2\right]_0^6 \left(= 4\lambda\pi(72+36) = 432\lambda\pi\right)$	A1	1.1b
	Distance from $O = \frac{\text{their } 432\lambda\pi}{120\lambda\pi}$	DM1	3.1b
	$= \frac{432}{120} = 3.6(\text{cm})$ *	A1*	2.2a
		(4)	
6c	Use of $\frac{3}{8}\times 3$	B1	1.2
	Moments about a diameter of the base	M1	3.1b
	$120\lambda\pi\times 3.6 + \left(6+\frac{3}{8}\times 3\right)\times\frac{2}{3}\pi(3)^3\lambda = (120+18)\pi\lambda d$	A1 A1	1.1b 1.1b
	$\left(d = \frac{747}{184} = 4.0597...\right)$		
	$\tan\alpha° = \frac{2}{\text{their } d}$	M1	2.1
	$\alpha = 26.2$	A1	1.1b
		(6)	
			Total 13 marks

Notes:

6a	M1	Correct method for total mass
	A1	Correct integration with limits seen or implied
	A1*	Obtain given answer from correct working
6b	M1	Correct method for moments about y-axis (condone missing π and λ)

	A1	Correct unsimplified integral
	DM1	Correct method to obtain distance from O; dependent on first M.
	A1*	Obtain given answer from correct working
6c	B1	Use of correct formula for c of m of a hemisphere, seen or implied
	M1	Condone use of a parallel axis. Require relevant terms and dimensionally correct. Condone common factors cancelled throughout
	A1 A1	Unsimplified equation with at most one error. Incorrect volume of hemisphere is only one error Correct unsimplified equation
	M1	Correct use of trigonometry to obtain α
	A1	$26\left(26.2265...\right)$ or better

Question	Scheme	Marks	AOs
7a			
	Energy equation	M1	3.1b
	$\frac{1}{2}mU^2 + mgr(1-\cos\theta) = \frac{1}{2}mv^2$	A1	1.1b
	$v^2 = U^2 + 2gr(1-\cos\theta)$	A1	1.1b
		(3)	
7b	Equation for circular motion at B	M1	2.1
	$mg\cos\theta - R = \frac{mW^2}{r}$	A1	1.1b
	Use $R=0$: $mg\cos\theta = \frac{m\left(U^2 + 2gr(1-\cos\theta)\right)}{r}$	M1	3.3
	$rmg\cos\theta = m\left(\frac{2rg}{3} + 2gr(1-\cos\theta)\right)$ $\left(\Rightarrow \cos\theta = \frac{8}{9}\right)$	A1	1.1b
	$\Rightarrow W^2 = rg\cos\theta = \frac{8}{9}rg$ *	A1*	2.2a
		(5)	
7c	Energy equation	M1	3.1b
	From B: $\frac{1}{2}mV^2 = \frac{1}{2}mW^2 + mgr\cos\theta$ or from A: $\frac{1}{2}mV^2 = \frac{1}{2}mU^2 + mgr$	A1	1.1b
	$V^2 = v^2 + 2gr\cos\theta = \frac{8rg}{9} + \frac{16rg}{9} = \frac{8rg}{3}$, $V = \sqrt{\frac{8rg}{3}}$	A1	1.1b
		(3)	
7d			

	Form an equation in α	M1	3.1b
	$\cos\alpha° = \frac{W\cos\theta}{V}\left(= \frac{\frac{8}{9}\sqrt{\frac{8rg}{9}}}{\sqrt{\frac{8rg}{3}}} = \frac{8\sqrt{3}}{27}\right)$	A1ft	1.1b
	$\alpha = 59(.12276...)$	A1	1.1b
		(3)	
			Total 14 marks

Notes:		
7a	M1	Equation for conservation of mechanical energy. All terms required and no extras; dimensionally correct. Condone sine / cosine confusion and sign errors.
	A1	Correct unsimplified equation
	A1	Or equivalent with v^2 as subject
7b	M1	Equation for circular motion. Dimensionally correct. Condone sine / cosine confusion and sign errors. Condone if $R = 0$ seen or implied at this stage.
	A1	Correct unsimplified equation
	M1	Use $R = 0$ in a relevant equation to obtain equation in r, g and $\cos\theta$ or W
	A1	Any correct equation in r, g and $\cos\theta$ or r, g and W E.g. $W^2 = \frac{2gr}{3} + 2gr - 2W^2$
	A1*	Obtain given result from correct exact working
7c	M1	**Complete** method to find the speed, e.g. by using conservation of energy or projectile motion. All terms required and no extras; dimensionally correct.
	A1	Correct unsimplified equation(s)
	A1	Any equivalent form.
7d	M1	Complete method to form a trig ratio for α
	A1ft	Correct use of their values to obtain a ratio for α. Ft their V. $\left(\sin\alpha = \frac{\sqrt{537}}{27} = 0.858, \quad \tan\alpha = \frac{\sqrt{179}}{8} = 1.67\right)$
	A1	59 or better

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